Week 4 Video 3

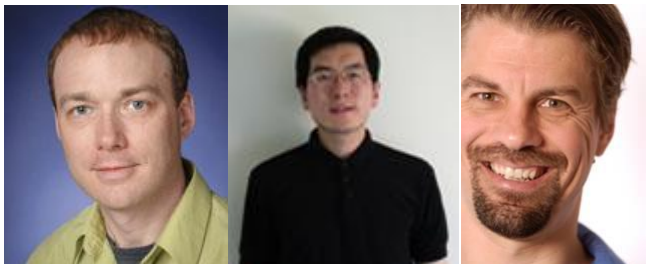
Knowledge Inference: Performance Factors Analysis

Performance Factors Analysis

- An alternative to BKT

- Addresses some of the limitations of BKT
- But doesn't have all of the nice features of BKT

- Proposed in 2009 by Pavlik, Cen, & Koedinger



PFA

- Measures how much latent skill a student has, while they are learning
  - But expresses it in terms of probability of correctness, the next time the skill is encountered
  - No direct expression of the amount of latent skill, except this probability of correctness

What is the typical use of PFA?

- Assess a student's knowledge of topic X
- Based on a sequence of items that are dichotomously scored
  - E.g. the student can get a score of 0 or 1 on each item
- Where the student can learn on each item, due to help, feedback, scaffolding, etc.

How does PFA differ from BKT?

Key assumptions

- Each item may involve multiple latent skills or knowledge components
  - Different from BKT

- Each skill has success learning rate γ and failure learning rate ρ
  - Different from BKT where learning rate is the same, success or failure

Key assumptions

- There is also a difficulty parameter β, but its semantics can vary – more on this later

- From these parameters, and the number of successes and failures the student has had on each relevant skill so far, we can compute the probability P(*m*) that the learner will get the item correct

PFA

PFA

$$\mathrm{m}(i, j \in KCs, k \in Items, s, f) = \beta + \sum_{j \in KCs} (\gamma_j s_{i,j} + \rho_j f_{i,j})$$

$$\mathrm{p}(m) = \frac{1}{1 + e^{-m}}$$

Reasonable Example

- $\gamma = 0.2$, $\rho = 0.1$, $\beta = -0.5$

Actual	m	P(m)

Reasonable Example

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Actual	m	P(m)
	-0.5	0.38

Reasonable Example

- $\gamma = 0.2$, $\rho = 0.1$, $\beta = -0.5$

Actual	m	P(m)
0	-0.5	0.38
	-0.5+(0.1*1)	

Reasonable Example

- $\gamma = 0.2$, $\rho = 0.1$, $\beta = -0.5$

Actual	m	P(m)
0	-0.5	0.38
	-0.5+0.1	

Reasonable Example

- $\gamma = 0.2$, $\rho = 0.1$, $\beta = -0.5$

Actual	m	P(m)
0	-0.5	0.38
	-0.4	0.40

Reasonable Example

- $\gamma = 0.2$, $\rho = 0.1$, $\beta = -0.5$

Actual	m	P(m)
0	-0.5	0.38
0	-0.4	0.40

Reasonable Example

- $\gamma = 0.2$, $\rho = 0.1$, $\beta = -0.5$

Actual	m	P(m)
0	-0.5	0.38
0	-0.4	0.40
	-0.5+(0.1*2)	

Reasonable Example

- $\gamma = 0.2$, $\rho = 0.1$, $\beta = -0.5$

Actual	m	P(m)
0	-0.5	0.38
0	-0.4	0.40
	-0.5+0.2	

Reasonable Example

- $\gamma = 0.2$, $\rho = 0.1$, $\beta = -0.5$

Actual	m	P(m)
0	-0.5	0.38
0	-0.4	0.40
	-0.3	

Reasonable Example

- $\gamma = 0.2$, $\rho = 0.1$, $\beta = -0.5$

Actual	m	P(m)
0	-0.5	0.38
0	-0.4	0.40
	-0.3	0.43

Reasonable Example

- $\gamma = 0.2$, $\rho = 0.1$, $\beta = -0.5$

Actual	m	P(m)
0	-0.5	0.38
0	-0.4	0.40
1	-0.3	0.43

Reasonable Example

- $\gamma = 0.2$, $\rho = 0.1$, $\beta = -0.5$

Actual	m	P(m)
0	-0.5	0.38
0	-0.4	0.40
1	-0.3	0.43

-0.5+(0.1*2)+(0.2*1)

Reasonable Example

- $\gamma = 0.2$, $\rho = 0.1$, $\beta = -0.5$

Actual	m	P(m)
0	-0.5	0.38
0	-0.4	0.40
1	-0.3	0.43

-0.5+0.2+0.2

Reasonable Example

- $\gamma = 0.2$, $\rho = 0.1$, $\beta = -0.5$

Actual	m	P(m)
0	-0.5	0.38
0	-0.4	0.40
1	-0.3	0.43
	-0.1	0.48

Degenerate Example

- $\gamma = 0.1$, $\rho = 0.2$, $\beta = -0.5$

Actual	m	P(m)
0	-0.5	0.38
0	-0.3	0.43
1	-0.1	0.48
	0	0.5

“Negative Learning”

- $\gamma = 0.1$, $\rho = -0.5$, $\beta = -0.5$

Actual	m	P(m)
0	-0.5	0.38
0	-1	0.27
1	-1.5	0.18
	-1.4	0.20

Note

- Values of ρ below 0 don't actually mean negative learning
- They mean that failure provides more evidence on lack of knowledge
- Than the learning opportunity causes improvement

Note

- Parameters in PFA combine information from correctness with improvement from practice improvement
- Makes PFA models a little harder to interpret than BKT

Adjusting β

- γ = 0.2, ρ = 0.1, β = -0.5

Actual	m	P(m)
0	-0.5	0.38
0	-0.4	0.40
1	-0.3	0.43
	-0.1	0.48

Adjusting β

- $\gamma = 0.2$, $\rho = 0.1$, $\beta = -1.5$

Actual	m	P(m)
0	-1.5	0.18
0	-1.4	0.20
1	-1.3	0.21
	-1.1	0.25

Adjusting β

- γ = 0.2, ρ = 0.1, β = +3.0

Actual	m	P(m)
0	3.0	0.953
0	3.1	0.957
1	3.2	0.961
	3.4	0.968

β Parameters

- Pavlik proposes three different β Parameters
  - Item
  - Item-Type
  - Skill

- Result in different number of parameters
  - And greater or lesser potential concern about over-fitting

Fitting PFA

- Typically Expectation Maximization is used
- Same algorithm as used in BNT-SM and Fit BKT at Scale

Expectation Maximization

1. Starts with initial values for each parameter
2. Estimates student correctness at each problem step
3. Estimates params using student correctness estimates
4. If goodness is substantially better than last time it was estimated, and max iterations has not been reached, go to step 2

Expectation Maximization

- EM is vulnerable to local minima
- Randomized restart typically used

Is it better?

- Jury is still out on whether PFA is better than BKT for cases where both apply

PFA .vs. BKT

- PFA beats BKT
  - Pavlik, Cen, & Koedinger (2009)
    - ΔA' = 0.01, 0.01, 0.02, 0.02
  - Gong, Beck, & Heffernan (2010)
    - ΔA' = 0.01
  - Pardos, Baker, Gowda, & Heffernan (2012)
    - ΔA' = 0.02
- BKT beats PFA
  - Baker, Pardos, Gowda, Nooraei, & Heffernan (2011)
    - ΔA' = 0.03
  - Pardos, Gowda, Baker, & Heffernan (2011)
    - (Predicting post-test)
    - Δr = 0.24, ΔRMSE = 0.01

Final Thoughts

- PFA is a competitor for measuring student skill, which predicts the probability of correctness rather than latent knowledge
- Can handle multiple KCs for the same item, a big virtue

Next Up

- Item Response Theory