# Week 5 Video 1

Relationship Mining

Correlation Mining

# Relationship Mining

- Discover relationships between variables in a data set with many variables

- Many types of relationship mining

# Correlation Mining

- Perhaps the simplest form of relationship mining

- Finding substantial linear correlations between variables
  - Remember this from earlier in the class?

- In a large set of variables

# Use Cases

- You have 100 variables, and you want to know how each one correlates to a variable of interest
  - Not quite the same as building a prediction model
- You have 100 variables, and you want to know how they correlate to each other

# Many Uses…

- Studying relationships between questionnaires on traditional motivational constructs (goal orientation, grit, interest) and student reasons for taking a MOOC
- Correlating features of the design of mathematics problems to a range of outcome measures
- Correlating features of schools to a range of outcome measures

# The Problem

- You run 100 correlations (or 10,000 correlations)
- 9 of them come up statistically significant
- Which ones can you "trust"?

# If you…

- Set p=0.05

- Then, assuming just random noise
- 5% of your correlations will still turn up statistically significant

# The Problem

- Comes from the paradigm of conducting a single statistical significance test

# The Solution

- Adjust for the probability that your results are due to chance, using a *post-hoc control*

# Two paradigms

- **FWER – Familywise Error Rate**
  - Control for the probability that any of your tests are falsely claimed to be significant (Type I Error)

- **FDR – False Discovery Rate**
  - Control for the overall rate of false discoveries

# Bonferroni Correction

- The classic approach to FWER correction is the Bonferroni Correction

# Bonferroni Correction

- Ironically, derived by Miller rather than Bonferroni

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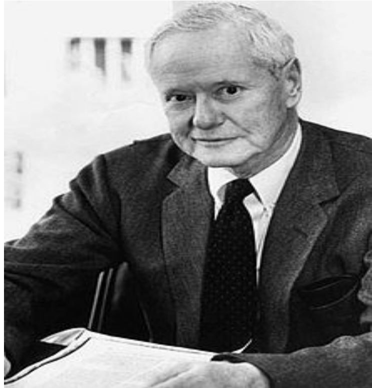
- Also ironically, there appear to be no pictures of Miller on the internet

# Bonferroni Correction

- A classic example of Stigler's Law of Eponomy
  - "No scientific discovery is named after its original discoverer"

# Bonferroni Correction

- A classic example of Stigler's Law of Eponomy
  - "No scientific discovery is named after its original discoverer"
  - Stigler's Law of Eponomy was proposed by Robert Merton



# Bonferroni Correction

- If you are conducting *n* different statistical tests on the same data set

- Adjust your significance criterion $\alpha$ to be
  - $\alpha$ / n

- E.g. For 4 statistical tests, use statistical significance criterion of 0.0125 rather than 0.05

# Bonferroni Correction: Example

- Five tests
  - $p=0.04$, $p=0.12$, $p=0.18$, $p=0.33$, $p=0.55$
- Five corrections
  - All p compared to $\alpha = 0.01$
  - None significant anymore
  - $p=0.04$ seen as being due to chance

# Bonferroni Correction: Example

- Five tests
  - $p=0.04$, $p=0.12$, $p=0.18$, $p=0.33$, $p=0.55$

- Five corrections
  - All p compared to $\alpha= 0.01$
  - None significant anymore
  - $p=0.04$ seen as being due to chance
  - Does this seem right?

# Bonferroni Correction: Example

- Five tests
  - p=0.001, p=0.011, p=0.02, p=0.03, p=0.04
- Five corrections
  - All p compared to $\alpha$= 0.01
  - Only p=0.001 still significant

# Bonferroni Correction: Example

- Five tests
  - p=0.001, p=0.011, p=0.02, p=0.03, p=0.04

- Five corrections
  - All p compared to $\alpha = 0.01$
  - Only p=0.001 still significant
  - Does this seem right?

# Quiz

- If you run 100 tests, which of the following are statistically significant?

A) 0.05
B) 0.01
C) 0.005
D) 0.001
E) All of the Above
F) None of the Above

# Bonferroni Correction

- Advantages
  - You can be “certain” that an effect is real if it makes it through this correction
  - Does not assume tests are independent
    - In our “100 correlations with the same variable” case, they aren’t!

- Disadvantages
  - Massively over-conservative
  - Throws out everything if you run a lot of correlations

# Often attacked these days

- Arguments for rejecting the sequential **Bonferroni** in ecological studies. MD Moran - Oikos, 2003 - JSTOR
- Beyond **Bonferroni**: less conservative analyses for conservation genetics. SR Narum - Conservation Genetics, 2006 – Springer
- What's wrong with **Bonferroni** adjustments. TV Perneger - Bmj, 1998 - bmj.com
- p Value fetishism and use of the **Bonferroni** adjustment. JF Morgan - Evidence Based Mental Health, 2007

# There are FWER corrections that are a little less conservative…

- Holm Correction/Holm's Step-Down (Toothaker, 1991)
- Tukey's HSD (Honestly Significant Difference)
- Sidak Correction

- Still generally very conservative
- Lead to discarding results that probably should not be discarded

# FDR Correction

- (Benjamini & Hochberg, 1991)

# FDR Correction

- Different paradigm, arguably a better match to the original conception of statistical significance

# Statistical significance

- $p<0.05$
- A test is treated as rejecting the null hypothesis if there is a probability of under 5% that the results could have occurred if there were only random events going on
- This paradigm accepts from the beginning that we will accept junk (e.g. Type I error) 5% of the time

# FWER Correction

- $p<0.05$

- Each test is treated as rejecting the null hypothesis if there is a probability of under 5% divided by N that the results could have occurred if there were only random events going on

- This paradigm accepts junk far less than 5% of the time

# FDR Correction

- $p<0.05$

- Across tests, we will attempt to accept junk exactly 5% of the time
  - Same degree of conservatism as the original conception of statistical significance

# FDR Procedure (Benjamini & Hochberg, 1991)

- Order your n tests from most significant (lowest p) to least significant (highest p)
  - Test your first test according to significance criterion $\alpha*1$ / n
  - Test your second test according to significance criterion $\alpha*2$ / n
  - Test your third test according to significance criterion $\alpha*3$ / n
  - Quit as soon as a test is not significant

# FDR Correction: Example

- Five tests
  - p=0.001, p=0.011, p=0.02, p=0.03, p=0.04

# FDR Correction: Example

- Five tests
  - p=0.001, p=0.011, p=0.02, p=0.03, p=0.04

- First correction
  - $p = 0.001$ compared to $\alpha = 0.01$
  - Still significant!

# FDR Correction: Example

- Five tests
  - $p=0.001$, $p=0.011$, $p=0.02$, $p=0.03$, $p=0.04$

- Second correction
  - $p = 0.011$ compared to $\alpha = 0.02$
  - Still significant!

# FDR Correction: Example

- Five tests
  - p=0.001, p=0.011, p=0.02, p=0.03, p=0.04
- Third correction
  - p = 0.02 compared to $\alpha$= 0.03
  - Still significant!

# FDR Correction: Example

- Five tests
  - p=0.001, p=0.011, p=0.02, p=0.03, p=0.04
- Fourth correction
  - p = 0.03 compared to $\alpha$= 0.04
  - Still significant!

# FDR Correction: Example

- Five tests
  - p=0.001, p=0.011, p=0.02, p=0.03, p=0.04

- Fifth correction
  - p = 0.04 compared to $\alpha$= 0.05
  - Still significant!

# FDR Correction: Example

- Five tests
  - $p=0.04$, $p=0.12$, $p=0.18$, $p=0.33$, $p=0.55$

# FDR Correction: Example

- Five tests
  - $p=0.04$, $p=0.12$, $p=0.18$, $p=0.33$, $p=0.55$

- First correction
  - $p = 0.04$ compared to $\alpha = 0.01$
  - Not significant; stop

# Conservatism

- Much less conservative than Bonferroni Correction
- Much more conservative than just accepting $p<0.05$, no matter how many tests are run

# q value extension in FDR (Storey, 2002)

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- p = probability that the results could have occurred if there were only random events going on

- q = probability that the current test is a false discovery, given the post-hoc adjustment

# q value extension in FDR (Storey, 2002)

- q can actually be lower than p
- In the relatively unusual case where there are many statistically significant results

# Closing thought

- Correlation mining can be a powerful way to see what factors are mathematically associated with each other
- Important to get the right level of conservatism

# Next lecture

- Causal mining